

**Revealing intermittency in nuclear multifragmentation  
with  $4\pi$  detectors**

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## Abstract

*The distortion on the intermittency signal, due to detection efficiency and to the presence of pre-equilibrium emitted particles, is studied in a schematic model of nuclear multi-fragmentation. The source of the intermittency signal is modeled with a percolating system. The efficiency is schematized by a simple function of the fragment size, and the presence of pre-equilibrium particles is simulated by an additional non-critical fragment source. No selection on the events is considered, and therefore all events are used to calculate the moments. It is found that, despite the absence of event selection, the intermittency signal is quite resistant to the distortion due to the apparatus efficiency, while the inclusion of pre-equilibrium particles in the moment calculation can substantially reduce the strength of the signal. Pre-equilibrium particles should be therefore carefully separated from the rest of the detected fragments, before the intermittency analysis on experimental charge or mass distributions is carried out.*

In high energy proton–nucleus and nucleus–nucleus collisions, as well as in heavy–ion reactions at intermediate energies, events with very large charge particle multiplicities are observed. The theoretical analysis of these events is hindered by the complexity of the reaction mechanisms, and often biased by model dependent assumptions. In the experimental data large fluctuations in various physical quantities are apparent, either in each event, either from one event to another. In recent years it was realized [1-3] that these fluctuations are quite useful for the study of the nuclear dynamics and the reaction mechanisms. They can be analyzed in terms of global variables, whose trends can be used to characterize high multiplicity events in an essentially model independent way. In particular, the intermittency analysis [2,3] has proven to be one of the most powerful and promising method to this respect. Intermittency patterns have been experimentally observed, both for rapidity charge particle distributions in high energy proton–nucleus collisions [2], and for fragment charge distributions in nuclear emulsion data on projectile multifragmentation in peripheral heavy–ion collisions at energies around 1 GeV/A [4].

The intermittency analysis of the fluctuations is essentially a multifractal analysis of a given distribution. This method has been developed in many fields of physics, ranging from hydrodynamics to astrophysics. For the problem in exam, the intermittency analysis is performed by studying the behaviour of the moments of the distribution, as one varies the resolution  $\delta s$  with which the distribution itself is considered. Only event to event fluctuations will be considered in this work, therefore the definition of the so called scaled factorial moments  $F_i$  which will be used, reads

$$F_i = \frac{\sum_k \langle n_k(n_k - 1) \dots (n_k - i + 1) \rangle}{\sum_k \langle n_k \rangle^i} \quad (1)$$

For a fixed resolution  $\delta s$ , the interval of  $s$  values is divided in bins of size  $\delta s$ . Then  $n_k$  is the number of fragments in the  $k$ -th bin for a given event, and the average is performed on the considered set of events. A diverging power law behaviour of these moments with decreasing resolution is, by definition, the intermittency signal, namely

$F_i \sim (\delta s)^{-\lambda_i}$ ,  $\delta s \rightarrow 0$ . In a plot of  $\log F_i$  versus  $-\log \delta s$ , this corresponds to the linear rise of the logarithms of the factorial moments. The corresponding slopes  $\lambda_i$  are called the intermittency indices, one for each order  $i$ . For a discrete and limited distribution, like the mass or charge distributions, this behaviour is of course demanded only in the limited range of physically admissible values of  $\delta s$ , in particular  $\delta s$  cannot be smaller than 1.

In this work we consider the problem of the disturbances on the intermittency signal which can be produced either by the inefficiencies of the detection apparatus, either by the reaction mechanism. In fact, the detection of the intermittency signal could be strongly reduced, or even suppressed, by the finite efficiency of the detection apparatus. This is of particular interest for the experimental works with  $4\pi$ -detectors for heavy-ion collisions at intermediate energy, which have been recently put in operation or are planned for the near future. A similar analysis has been presented in ref. [5] for other global dynamical variable, like the transverse momentum, the flow angle, and so on.

On the other hand, the source of the intermittency signal is believed to be associated with the critical behaviour of the nuclear system [3], either due to a phase transition or to a mechanical instability [3,6]. However, in general one can expect that only a part of the nuclear system reaches the critical region, and therefore that the intermittency signal can be clearly detected only if the fragments coming from the participating zone are properly selected.

The simplest model which produces an intermittency signal, associated with a phase transition, is the percolation model [3,7]. It has been widely used [8] in the analysis of the experimental data on mass and charge distributions in heavy ion reactions. In this work we assume that the source of the intermittency signal in the mass/charge distribution can be modeled by a percolating system, and we study the effects on this signal of various detection inefficiencies, as well as of the presence of another non-critical source. The results of this analysis should not be dependent on the particular model, which is used to generate the intermittency signal.

The apparatus inefficiency is schematized in a simple way, by assigning to each fragment of size  $s$  a probability  $Prob(s)$  to be actually detected, and therefore a probability  $1 - Prob(s)$  to escape undetected from the apparatus. In a given event generated by the computer, for each cluster of the percolation model, which is assumed to represent a nuclear fragment, a random number  $x$ , belonging to the interval  $(0,1)$ , is generated and compared with  $Prob(s)$ . If  $x \leq Prob(s)$ , the fragment is assumed to have been detected, otherwise it is assumed to have passed undetected. This global parameterization of the efficiency can, in particular, roughly represent the geometrical efficiency of the apparatus. Of course the particular form of the probability distribution  $Prob(s)$  depends, in this case, not only on the geometry of the detector, but also on the reaction dynamics, since in general fragments of different size are emitted preferentially in different directions, where the geometrical efficiency can be rather different. However, in this paper we are not interested in any particular detector or heavy-ion reactions, but rather to establish general trends, which can be used as a guidance to the inefficiency effects that one can expect. Therefore, the function  $Prob(s)$  will be used to represent globally the overall efficiency of the apparatus, including the geometrical efficiency, the detection efficiency, the threshold cuts, and so on.

In the nuclear emulsion data [4], where the intermittency signal was observed, the charge of each fragment of the projectile multifragmentation was identified, and only the events for which the total sum of the fragment charges was equal to the projectile charge were included in the analysis. Because of the nuclear emulsion efficiency, this selection can bias the final result. However, simulations with a model [6], which successfully reproduces the intermittency signal, show that in these cases the bias does not affect the main trends. With  $4\pi$  detectors for heavy ion collisions at intermediate energy, the average total efficiency  $\overline{Prob}$  can hardly exceed 0.7–0.8. With average charged fragment multiplicities of the order 40 to 60, to maintain such a strong selection of the events would imply a severe reduction of the counting rate, even down to the limits of the experimental feasibility, in

the worst cases. It is customary, for this reason, to apply a less stringent selection on the events, usually by only demanding [5] that the total sum of the observed fragment charges is not less than 90% of the initial total charge, or so. In this paper we will adopt the extreme attitude of excluding any selection on the events, so that all the events will be used in calculating the moments. This means that, even if a few fragments are missing in a given event after applying the efficiency filter  $Prob(s)$ , the detected fragment distribution will still be used for calculating the numerator and the denominator of eq. (1) and the corresponding contribution of the event to the average value. It is expected that the introduction of a proper selection on the events would result in an enhancement of the intermittency signal.

In the simulation we will consider, for simplicity, only one type of site in the percolating system, and therefore the size variable  $s$  can be interpreted as the charge or the mass of the fragment, according to the problem under study. This oversimplification does not affect the analysis that will be presented.

As a side remark, it has to be noticed that in computer simulations much care has to be taken with the random generator routine, which can present undesired spurious correlations between calls. The routine used in the present work was checked under various statistical tests [9]. Furthermore, care must be taken about the convergence of the results with the numbers of events. The results here presented were obtained with the standard number of events equal to 30000. We checked that for a larger number of events the results are essentially unaffected, while for a smaller number the results can show fluctuations.

The intermittency signal can be generated in a percolating system by choosing the values of the site probability  $p$  and of the bond probability  $q$  just equal to the critical values  $p_c$  and  $q_c$  for the corresponding infinite system, in agreement with the general rule that intermittency is associated with critical points. In general it is considered more realistic to assume the parameters  $p$  and  $q$  to vary randomly in some range of values, since in the experimental situation events with different nature are usually mixed together. In this case

intermittency appears whenever the range of allowed random values includes the critical ones. This point is illustrated in the case of a  $6^3$  cubic lattice in Fig. 1, where, for  $p = 1$ , three intervals of uniformly distributed values of  $q$  are considered. As expected, the linear rise of the factorial moments is present only for the interval  $0.2 < q < 0.3$ , in agreement with the critical value  $q_c = 0.23$ , at  $p = 1$ , in the infinite system. This particular system will be considered in the sequel as the source of the intermittency signal. It is characterized by the intermittency indices  $\lambda_2 = 1.73 \cdot 10^{-3}$ ,  $\lambda_3 = 4.78 \cdot 10^{-3}$  and  $\lambda_4 = 1.02 \cdot 10^{-2}$ . The relevance of event mixing was also emphasized recently in ref. [10], where intermittency was observed only in a particular dynamical regime of the expanding emitting source model [11] of multifragmentation. In general mixing of events enhances the intermittency signal, if it is present, by increasing the values of the critical indices.

The efficiency function  $Prob(s)$  can be quite different for different detectors. For some experimental apparatus the efficiency is increasing with the size  $s$ , and it can be very low at small  $s$  values. The opposite situation is also possible, with some limit value of  $s$ , above which the fragments are undetected or unresolved. For a constant efficiency  $Prob(s) = \overline{Prob}$ , the intermittency signal appears essentially undisturbed, with a pattern hardly distinguishable from the one of Fig. 1b. We have checked that, even with a  $\overline{Prob}$  value as small as 0.5, the linear rise, and the corresponding slopes, remain mainly unaffected, despite the fact that on the average only 50% of the fragments are detected. A slightly more realistic case is considered in Figs. 2a–c, where the efficiency function is assumed to be of the form

$$Prob(s) = \begin{cases} P_1 + \frac{1-P_1}{s_0-1} \cdot (s-1) & \text{if } s \leq s_0 \\ 1 & \text{if } s > s_0 \end{cases} \quad (1)$$

where  $P_1$  is the efficiency for  $s = 1$ , and  $s_0$  is the value of  $s$  above which one assumes  $Prob(s) = 1$ . The latter has been fixed at  $s_0 = 10$  in the calculations of Fig. 2. To the extent that the value of  $P_1$  is not too small, we have found that the intermittency signal is only slightly disturbed, in a wide range of  $s_0$  values. Only when  $P_1$  is close to zero, the

linear rise of the factorial moments is destroyed. These results indicate that the detector efficiency must satisfy only mild conditions, in order to be sensitive to the intermittency signal. Essentially, in the specific schematic example, the condition  $P_1 \equiv Prob(1) \geq 0.2$  is required, without any stringent restriction on  $s_0$ .

If the efficiency  $Prob(s)$  is not too small, the frequency and their variance in each bin should be affected by approximately the same overall factor. Therefore the ratio of the numerator and denominator of Eq. (1) should be only slightly dependent on  $Prob(s)$ . Furthermore, the additional event-to-event fluctuations introduced by the efficiency filtering is multinomial in character, and they should be eliminated by the very definition of the factorial moments [2]. This argumentation could be one possible qualitative explanation for the stability of the signal, observed in the numerical simulations. Similar results have been obtained by assuming an efficiency  $Prob(s)$  decreasing with  $s$ . In particular we have studied the case with  $Prob(1) = 1$  and  $Prob(s)$  decreasing linearly with  $s$  down to zero for some value  $s_0$  of  $s$ . For  $s > s_0$ ,  $Prob(s)$  was considered to vanish identically. In this case apparent deviations from linear rise was observed only for quite small values of  $s_0$ , roughly for  $s_0 < 10$ .

We have also studied the effect of the mass/charge resolution of the detector. In general the resolution does not produce any distortion on the intermittency signal, even for unrealistically large values of the resolution width. The mass/charge resolution uncertainty, in fact, produces in each event a distortion of the mass/charge distribution, but it cannot change the intrinsic fractal nature of the distribution itself. This insensitivity to the resolution was checked in explicit numerical simulations, where, event by event, each fragment  $s$ -value was shifted randomly, according to a normal distribution of a given width, before calculating the factorial moments of Eq. (1).

The intermittency signal can be obscured or hidden not only by the detector inefficiencies, but also by the dynamics itself of the heavy-ion collisions. In particular, for non-central collisions, it is possible that some part of the nuclear system is only weakly



excited. It forms then the so-called "spectator" part of the reaction. This subsystem can however still emits a substantial fraction of the observed fragments, thus obscuring the intermittency signal which is expected to be originated by the "participating" part. This problem, however, can be avoided if one selects central collisions and reactions between nearly symmetrical systems.

Another possible physical disturbance, due to the reaction dynamics, is the presence of pre-equilibrium emitted particles, usually nucleons. By pre-equilibrium particles we mean generically particles that are emitted in the very preliminary stage of the reactions. They are usually quite energetic and preferentially go in the forward direction. At intermediate energies, according to the theoretical simulations, one expects several pre-equilibrium particles in all heavy-ion collisions. Their statistical properties should be completely different with respect to the ones of the particles emitted from the participating part. This is particularly true if, in the participating region, a phase transition occurs or an instability is present, like in a genuine multifragmentation process, for which intermittency should be present. The simplest way of introducing the pre-equilibrium particles in the previous schematic multifragmentation model is to add to the percolating system another independent particle source. The latter will be assumed to produce, on the average, a given number  $\overline{N}$  of particles ( $s = 1$ ), with fluctuations from one event to another which follow a gaussian distribution of a given width  $W$ . After adding, for each event, these particles to the fragment distribution produced by the percolating system, we perform again the same intermittency analysis and we check to which extent the signal is reduced. The results show that the disturbance depends mainly on the ratio  $R = W/\overline{N}$ , being larger if the ratio is smaller. A typical example is reported in Fig. 3, for  $\overline{N} = 20$  and different  $W$  values. For clarity of the figure only  $F_4$  is reported, but similar results are found for the other moments. This value of  $\overline{N}$  is about 25% of the average number of  $s=1$  fragments produced by the percolating system. When  $R \approx 0.5$ , namely the full width equal the average value, the intermittency signal starts to degrade, and it rapidly disappears for smaller values

of  $R$ . This result does not depend so much on  $\overline{N}$  itself, except when it is very small, of the order of few units, in which case the effect of the additional source become trivially negligible. The pre-equilibrium particles should be therefore identified in experiments in which the intermittency signal is looked for, and they should be systematically excluded in the calculation of the factorial moments. This is not always an easy task, since their energy spectra and angular distributions partly overlap in general with the ones pertinent to the system undergoing multifragmentation.

In summary we have presented a study of the disturbances on the intermittency signal that can be present in experiments with  $4\pi$  detectors for heavy-ion collisions at intermediate energy. The effects of both detector inefficiencies and reaction dynamics have been considered. While most of the  $4\pi$  detectors, that are already in operation, under construction, or planned, are expected to have an efficiency good enough to be sensitive to the intermittency signal, the main warning is about the presence of pre-equilibrium particles. The latter could cause, in fact, serious problems at the level of the phenomenological analysis, since they could mask completely the intermittency signal if they are not excluded in the calculation of the factorial moments.

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## Figure captions

Fig. 1. Scaled factorial moments for a  $6^3$  percolating lattice. The linear rise is observed only for case  $b$ ), for which the interval of random  $q$  values includes the critical value  $q_c = 0.23$ . This intermittency signal is the one used in the present analysis.

Fig. 2. The intermittency signal of Fig. 1b), after being filtered with the apparatus efficiency. For more details see the text.

Fig. 3. The intermittency signal obtained by adding pre-equilibrium particles (peps) to the source of Fig. 1b). The quantity  $W$  is the half-width of the peps number distribution. For more details see the text.